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Gravitational Radiation Evolution of Accreting Neutron Stars

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ABSTRACT

The gravitational-wave and accretion driven evolution of neutron stars in low mass X-ray binaries and similar systems is analyzed, while the amplitude of the radiating perturbation (here assumed to be an r-mode) remains small. If most of the star is superfluid, with (temperature independent) mutual friction dominating the ordinary (temperature dependent) shear viscosity, the amplitude of the mode and the angular velocity of the star oscillate about their equilibrium values with a period of at least a few hundred years. The resulting oscillation of the neutron star temperature is also computed. For temperature dependent viscosity, the general conditions for the equilibrium to be stable are found.

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1. Introduction

We shall study the evolution of rapidly rotating accreting neutron stars under the influence of their emission of gravitational radiation. We modify and extend the two-component model of the star (equilibrium plus perturbation) introduced by Owen et al. (1998) and also employed by Levin (1999), but restrict the analysis to small perturbations. These are assumed to be in the form of r-modes (Andersson 1998; Friedman & Morsink 1998; Lindblom, Owen & Morsink 1998; Andersson, Kokkotas & Schutz 1999), which radiate mainly via Coriolis-driven velocity perturbations rather than the density perturbations of the less powerful f-modes. We must allow for large uncertainties in many of the relevant properties of neutron stars, such as the superfluid transition temperature and the effects of magnetic fields and a core-crust boundary layer.

After developing a general formalism, we shall study the evolution of the accreting neutron star under conditions in which its angular velocity remains approximately constant. One reason for this restriction is our interest in conditions in which the gravitational radiation is persistent over long time scales. This requires the existence of a stable or (possibly) overstable equilibrium. We shall see under what conditions the system can evolve toward this equilibrium, in which the rate of accretion of angular momentum from the surrounding disk is balanced by its rate of loss via gravitational radiation. If this equilibrium is achieved, the observed flux of gravitational radiation can be shown to be proportional to the observed flux of X-rays from the accretion (Wagoner 1984; Bildsten 1998).

One of our longer term goals is the development of parameterized expressions describing possible time evolutions of the gravitational-wave frequency and amplitude, to facilitate detection by LIGO, VIRGO, and similar laser interferometer detectors. The brightest low mass X-ray binaries (LMXBs) are the prime targets.

2. Dynamical and Thermal Evolution

Consider a Newtonian neutron star in equilibrium (with equatorial radius R) which is perturbed by a nonaxisymmetric infinitesimal fluid displacement $\vec{\xi} = \vec{f}(r,\theta)e^{i(m\phi+\sigma t)} \sim \alpha R$, with $\alpha \ll 1$. Based on the work of Friedman & Schutz (1978a) and Levin & Ushomirsky (2001a), the total angular momentum J of the star can be decomposed into its equilibrium angular momentum J_* and a perturbation proportional to the canonical angular momentum J_c. That is,

$$J = J_*(M,\Omega) + (1-K_j)J_c \, , \qquad J_c = -K_c\alpha^2 J_* \, , \tag{1}$$

where M is the mass and Ω is the (uniform) angular velocity of the equilibrium star. All constants $K_{(\,)}$ will be dimensionless, with $K_j \sim K_c \sim 1$.

In classical mechanics, the action $I = E/\omega$ of any normal mode of a set of oscillators (with frequency ω) is an adiabatic invariant. For a fluid, the analogous quantity should be $\tilde{E}_c/\omega$, where $\tilde{E}_c$ is the canonical energy of the perturbation in the corotating frame and $\omega = \sigma + m\Omega$ is its frequency in that frame. However, we also have the general relation $\tilde{E}_c = -(\omega/m)J_c$ (Friedman & Schutz 1978a). Therefore, following Ho & Lai (2000), we assume that the canonical angular momentum is also an adiabatic invariant, and should therefore be unaffected by the slow rate of mass accretion. Thus it obeys the usual relation (Friedman & Schutz 1978b)

$$dJ_c/dt = 2J_c[(F_g(M,\Omega) - F_v(M,\Omega,T_v)]\ , \tag{2}$$

where F_g is the gravitational radiation growth rate and F_v is the viscous damping rate. The latter usually depends upon a spatially averaged temperature $T_v(t)$.

On the other hand, conservation of total angular momentum requires that

$$dJ/dt = 2J_cF_g + \dot{J}_a(t)\ , \tag{3}$$

where $\dot{J}_a = j_a\dot{M}$ is the rate of accretion of angular momentum. The mass is accreted with specific angular momentum j_a at a rate $\dot{M}(t)$.

Combining these equations then gives the dynamical evolution relations

$$\begin{aligned} \frac{1}{\alpha}\frac{d\alpha}{dt} &= F_g - F_v + [K_jF_g + (1-K_j)F_v]K_c\alpha^2 - \left(\frac{j_a}{2J_*}\right)\dot{M}(t)\ , && (4)\\ \left(\frac{I_*}{J_*}\right)\frac{d\Omega}{dt} &= -2[K_jF_g + (1-K_j)F_v]K_c\alpha^2 + \left[\frac{(j_a - j_*)}{J_*}\right]\dot{M}(t)\ ; && (5) \end{aligned}$$

where $I_*(M,\Omega) = \partial J_*/\partial\Omega$ and $j_*(M,\Omega) = \partial J_*/\partial M$.

In obtaining our thermal evolution relation, the large uncertainties in some thermodynamic properties of the neutron star make it sufficient to consider slowly rotating stars. Thermal energy conservation for the entire star then gives

$$\int \frac{\partial T}{\partial t} c_v dV \equiv C(T)\frac{dT}{dt} \cong 2\tilde{E}_cF_v(T_v) + K_n\langle\dot{M}\rangle c^2 - L_\nu(T_\nu)\ , \tag{6}$$

where the rotating frame canonical energy $\tilde{E}_c = K_e\Omega J_*\alpha^2$. Since the main contributor to the specific heat is the degenerate relativistic electrons, its value at constant volume (c_v) is essentially the same as that at constant pressure. The emissivities are produced by viscous

heating, pycnonuclear reactions and neutron emissions in the inner crust (proportional to a time-averaged mass accretion rate), and neutrino losses. The hydrogen/helium burning rate is assumed to be balanced by the surface emission of photons (Schatz et al. 1999), especially at the large accretion rate $\dot{M} = 10^{-8} M_\odot$ yr^{-1} that we shall use. The mass accretion rate can be estimated from accretion energy conservation. The photon luminosity arising directly from the accretion is $L_{acc} \approx (GM/R)\dot{M}(t)$, for a slowly rotating neutron star with a negligible magnetosphere.

We are interested in the evolution of neutron stars after they have been spun up to the point where the gravitational radiation growth rate has become equal to the viscous damping rate:

$$F_g(\Omega_0, M_0) = F_v(\Omega_0, M_0, T_0) \equiv F_0 \ , \tag{7}$$

so the evolution equation (2) vanishes. This equality defines our initial state. Before that time, we see from equation(2) that any intrinsic perturbation could not grow from its (infinitesimal) value α_{min}. The initial temperature T_0 is determined by the vanishing of equation (6), with the nuclear heating in the inner crust balanced by the neutrino emission (Brown 2000). Since we are only considering conditions in which $\alpha^2 \ll 1$, the properties $\Omega(t)$ and $M(t)$ evolve much slower than $\alpha(t)$ and $T(t)$. For any other property Q_* of the unperturbed star, let $Q_0 \equiv Q_*(M_0, \Omega_0)$.

From now on we shall take the perturbation to be due to the dominant $l = m = 2$ r-mode. In order to facilitate comparison with previous results, we shall adopt the neutron star model of Owen et al. (1998) for numerical work. Then the gravitational radiation growth rate of this mode is

$$F_g = \tilde{\Omega}^6/\tau_{gr} \ , \quad \tau_{gr} = 3.26 \text{ sec} \ , \qquad \tilde{\Omega} \equiv \Omega(\pi G\langle\rho\rangle)^{-1/2} \ . \tag{8}$$

In the temperature range of interest ($10^8 < T < 10^{10}$ K), the viscous damping rate of this mode is approximated as

$$F_v \cong \left(\frac{\tilde{\Omega}^5}{\tau_{mf}}\right) e^{-(T_v/T_c)^2} + \frac{1}{\tau_{sh}} \left(\frac{10^9 \text{ K}}{T_v}\right)^2 \ , \tag{9}$$

where T_c is the superfluid transition temperature.

The first term is the contribution from the mutual friction between the neutron superfluid and the superconducting proton–relativistic electron fluid (Lindblom & Mendell 2000). Its behavior (and that of other properties considered below) as the temperature passes through the superfluid transition temperature T_c is approximated by the exponential. Although Lindblom & Mendell (2000) found that $\tau_{mf} \lesssim 10^4$ sec, we shall keep it as a free parameter because of the many uncertainties involved in the physics of this system, especially

when including magnetic effects such as the interaction of the vortex lines and flux tubes (Ruderman 2000). In fact, we fix it to satisfy our initial condition (7), when observationally relevant values of Ω_0, M_0, T_0 are chosen. To approximately match an inferrred maximum spin rate of 330 Hz for the neutron stars in LMXBs, we shall choose $\tilde{\Omega}_0 = \Omega_0(\pi G\langle\rho\rangle)^{-1/2} = 0.25$ (Levin 1999), which then fixes $\tau_{mf} \approx 13$ sec if $T \ll T_c$. The maximum rotation rate of a neutron star corresponds to $\tilde{\Omega} \cong 2/3$.

The second term is the contribution of the ordinary shear viscosity. Lindblom & Mendell (2000) obtained $\tau_{sh} = 1.0 \times 10^8$ sec for their superfluid neutron star model, dominated by electron–electron scattering in the superfluid regions and neutron–neutron scattering in the normal regions. Above the superfluid transition temperature, Lindblom, Owen & Morsink (1998) obtained $\tau_{sh} = 2.5 \times 10^8$ sec. There are also large uncertainties in this contribution, due to the shear in a boundary layer between the core and crust (Andersson, Jones, Kokkotas & Stergioulas 1999; Bildsten & Ushomirsky 2000; Lindblom, Owen & Ushomirsky 2000; Rieutord 2000; Wu, Matzner & Arras 2001) and the uncertain response of the crust to the mode (Levin & Ushomirsky 2001b).

In keeping with the fact that we are only working to lowest order in $\tilde{\Omega}$ (as well as the relativity parameter GM/Rc^2), we take $j_a - j_0 \cong j_a$ and $J_0 \cong I_0\Omega_0$. We also note that $K_c = 3K_e = 0.094$. (The value of K_j is unimportant.)

Now that we have specified all properties in the equations (4) and (5) of evolution of $\alpha(t)$ and $\Omega(t)$, we can consider the thermal evolution. In what follows we shall assume that thermal conductivity timescales are short enough to give relations $T_v(T)$ and $T_\nu(T)$ between these three spatially averaged temperatures that appear in equation (6). The normal and superfluid contribution to the specific heat and the neutrino luminosity that appear in this equation are approximated by

$$C(T) = [C'_{norm} e^{-(T_c/T)^2} + C'_{super}]T \qquad (C'_{norm} \cong 20 C'_{super}) \ , \tag{10}$$

$$L_\nu(T) = L'_{URCA} T^8 e^{-(T_c/T)^2} + L'_{brem} T^6 \ . \tag{11}$$

The constants L'_{URCA} and L'_{brem} are obtained by fitting the results of Brown (2000) for normal and superfluid neutron stars. We also take the nuclear heating constant $K_n = 1 \times 10^{-3}$ (Brown 2000).

In Figures 1 and 2 we show the results of integrating our three coupled evolution equations (4), (5), and (6), if $T_c \gg T_0$. The main feature is the spin-down (due to loss of angular momentum in gravitational waves) and viscous heating during the time when the amplitude α of the mode exceeds its equilibrium value (denoted by the dashed line).

For such large values of T_c, the evolution qualitatively resembles that which occurs

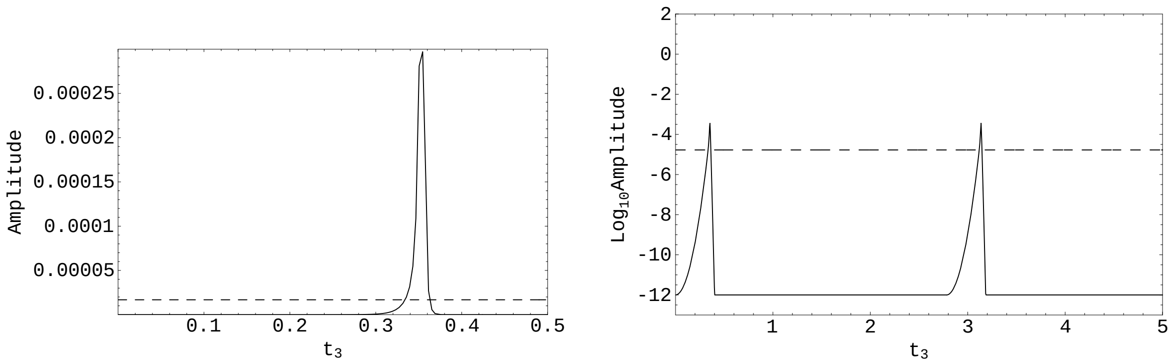

Fig. 1.— The evolution of the mode amplitude α, for the case $T_c \gg T_0$. The dashed line is its equilibrium value. Time is in units of 10^3 years. Subsequent cycles are similar.

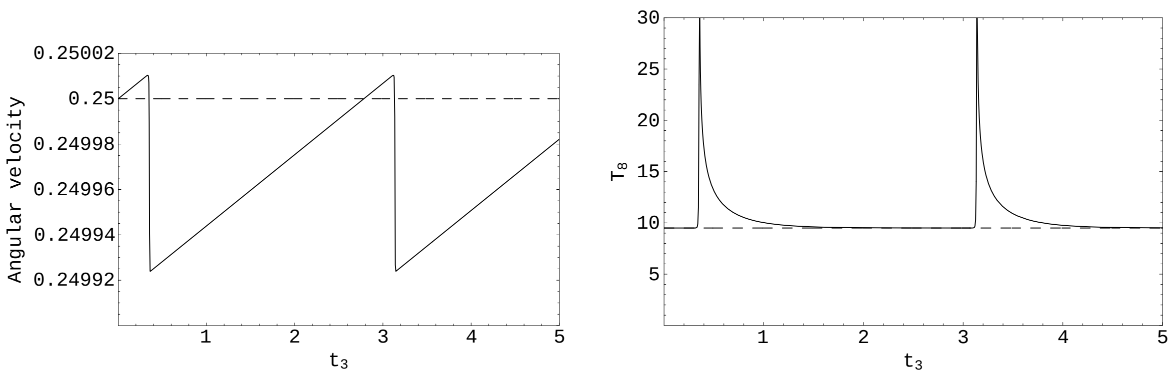

Fig. 2.— Same as Figure 1, for the angular velocity $\Omega(\pi G\langle\rho\rangle)^{-1/2}$ and the temperature $T/10^8$ K. Here the dashed line is the initial value.

when $\partial F_v/\partial T = 0$ (requiring neglect of the shear viscosity). In this case equations (4) and (5) decouple from equation (6). In addition, $x \equiv \ln\alpha$ completely decouples, obeying the equation

$$d^2x/dt^2 - 2K_cF_0e^{2x}dx/dt + dV/dx = 0 \;, \quad V(x) \cong F_0(K_cF_0e^{2x} - F_ax) \;, \tag{12}$$

where F_a is the time-averaged rate of accretion of angular momentum. The Eddington limit to the accretion rate gives $\tau_a \equiv 1/F_a \gtrsim 5 \times 10^6$ years. The sign of the damping term is opposite to that obtained by Levin (1999), leading to overstable oscillations about the equilibrium amplitude

$$\alpha_e \cong (F_a/2K_cF_0)^{1/2} \lesssim 2 \times 10^{-5}$$

with a period

$$P \cong [8\ln(\alpha_{max}/\alpha_{min})/(F_0F_a)]^{1/2} \gtrsim 300 \text{ years} \;.$$

The fraction $F_>$ of the time that $\alpha > \alpha_e$ is

$$F_> \cong \ln[8\ln(\alpha_{max}/\alpha_{min})]/4\ln(\alpha_{max}/\alpha_{min}) \sim 0.1 \; .$$

It is also found that α_{max} increases and α_{min} decreases on a timescale $\sim P/\alpha_{max}$. However, α_{min} is presumably limited by the intrinsic fluctuations in the neutron star, which is also why we have not allowed α to drop below its initial value in Figure 1. Because of the inclusion of the shear viscosity, the value of the period P in Figure 1 is about ten times longer than the above estimate while the value of the fraction $F_>$ in Figure 1 is about ten times less.

On the other hand, for values of $T_c \lesssim T_0$, the temperature dependence of the viscous damping rate F_v is strong enough to produce a thermal runaway to large values of T and α (outside the range of validity of our equations), as found by Levin (1999). This result is generalized in the next section.

3. Behavior Near Equilibrium

In contrast to the initial state, the equilibrium state X_e^i of our dynamical variables

$$X^i(t) = \{\alpha, \Omega, T\} = X_e^i[1 + \zeta^i(t)] \; , \qquad |\zeta^i| \ll 1 \; ,$$

is defined by the vanishing of the evolution equation (3), in addition to equations (2) and (6). Employing a constant (averaged) accretion rate, the evolution equations give

$$d\zeta^i/dt = A^{ij}\zeta^j \; , \qquad \zeta^i \propto \exp(\lambda t) \; , \quad ||A^{ij} - \lambda\delta^{ij}|| = 0 \; . \tag{13}$$

Assume now that $|\partial F_v/\partial T| \sim F_v/T_e$, etc.

The coefficients of the eigenvalue equation $\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0$ are

$$a_2 \cong \frac{1}{C_e}\left[\left(\frac{\partial L}{\partial T}\right)_e - 2(\tilde{E}_c)_e\left(\frac{\partial F_v}{\partial T}\right)_e\right] \sim K_r\alpha_e^2 F_0 \; , \tag{14}$$

$$a_1 \cong \frac{4(\tilde{E}_c)_e F_0}{C_e}\left(\frac{\partial F_v}{\partial T}\right)_e \sim K_r\alpha_e^2 F_0^2 \; , \tag{15}$$

$$\begin{aligned} a_0 \cong\; & \frac{4K_c\Omega_e\alpha_e^2 F_0}{C_e}\left[\frac{\partial(F_g - F_v)}{\partial\Omega}\right]_e\left(\frac{\partial L}{\partial T}\right)_e \\ & - \frac{16K_c(\tilde{E}_c)_e\alpha_e^2 F_0^2}{C_e}\left(\frac{\partial F_v}{\partial T}\right)_e \sim K_r\alpha_e^4 F_0^3 \; . \end{aligned} \tag{16}$$

The ratio of rotational to thermal energy is $K_r \equiv 2K_e\Omega_e J_0/C_eT_e \sim 10^5$. We have used the fact that the cooling rate $F_c \equiv L_\nu(T_e)/C_eT_e \sim K_r\alpha_e^2 F_0$.

Now we also employ the inequalities $K_r \gg 1$ and (mode energy)/(thermal energy) $\sim K_r \alpha_e^2 \lesssim 10^{-4} \Rightarrow |a_1| \gg a_2^2$ to obtain the eigenvalues

$$\lambda_{1,2} \cong -a_2/2 \pm \sqrt{-a_1}\ , \qquad \lambda_3 \cong -a_0/a_1\ .$$

We have used the fact that $|\lambda_3| \sim \alpha_e^2 F_0 \ll |\lambda_1| \sim |\lambda_2|$.

Let us examine the two relevant possibilities. For cases such as dominance by shear viscosity in equation (9), with $\tilde{E}_c > 0$,

$$a_1 \propto (\tilde{E}_c)_e (\partial F_v/\partial T)_e < 0 \implies \lambda_{1,2} \cong \pm\sqrt{-a_1} \sim K_r^{1/2} \alpha_e F_0\ .$$

Thus this equilibrium is unstable, with a growth rate λ_1 that is of the same magnitude as found by Levin (1999).

The other possibility is

$$a_1 > 0 \implies \lambda_{1,2} \cong -a_2/2 \pm i\sqrt{a_1}\ .$$

Thus stability also requires that $a_0 > 0$ and $a_2 > 0$. From their relations above, we see that this means that the variation in the cooling rate with temperature must be greater than twice the variation in the viscous heating rate with temperature (which is usually satisfied).

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